in: Liora Malki-Epsthein, Catalina Spataru, Ljiljana Marjanovic Halburd and Dejan Mumovic (Editors); 2014, "Proceedings of the 2014 Building Simulation and Optimization Conference"; 23-24 June 2014, UCL, London, UK

ECONOMICALLY EFFECTIVE AND ROBUST LOW-ENERGY DWELLINGS

Liesje Van Gelder, Hans Janssen and Staf Roels
Building Physics Section, KU Leuven, Leuven, Belgium

ABSTRACT

In building design, deterministic simulations are commonly used to calculate the building energy performance and corresponding costs. User behaviour, workmanship of building envelopes or services and future energy prices are however inherently uncertain and neglecting this variability may lead to excessive deviations between design and reality. To minimise these deviations, this paper suggests a probabilistic design method based on multi-layered Monte Carlo schemes, sensitivity analysis and meta-modelling. To obtain guidelines for economically effective and robust low-energy dwellings, this method is illustrated on the design of a semi-detached dwelling. The net present costs of the resulting optimal design options are effective and robust in all potential future user and economic scenarios.

INTRODUCTION

The energy efficiency of buildings is becoming increasingly important in view of the climate change and fuel depletion challenges. At present, new buildings should be low-energy, while passive and nearly zero energy buildings will become the standard in the near future. In the design of these buildings, deterministic simulations are commonly used to calculate the energy demand and related energy costs. User behaviour, workmanship of building envelopes or services and future energy prices are however inherently uncertain, and neglecting these uncertainties may lead to excessive deviations between design and reality. Such deviations are undesirable for the building owners, as they require confidence in the return on their investments in energy efficiency. Moreover, it is crucial for the environmental policy to accurately predict energy reductions. To minimise these deviations in order to support governments and convince the building owners, the development and promotion of economically effective and robust building envelopes and service solutions is an important step. Effectiveness, in this sense, is defined as the ability of the design option to optimise the performance, while robustness is defined as the ability to stabilise this performance for the entire range of input uncertainties.

The aim of this paper is thus to propose a probabilistic design method to obtain effective and robust building solutions. This method is then applied on a cost optimisation of a typical Flemish dwelling geometry in a social housing neighbourhood to illustrate how this can be used in decision-making.

First, the probabilistic design method is briefly explained. This is followed by the description of the case study to end with the analysis of the results.

PROBABILISTIC DESIGN METHOD

In optimisation problems, contributing input parameters can be divided into three categories, as shown in Figure 1. *Design parameters*, such as the intended air tightness, the type of ventilation system, …, are fully controllable and their values are to be selected in the design problem. Inherently *uncertain parameters*, such as the impact of workmanship, the actual ventilation rate value, ..., are variables, uncontrollable by the designer. Finally, *scenario parameters* are uncertain parameters dealing with future, for example economic or user, scenarios for which an explicit evaluation is asked. These parameter categories dictate to be ascribed to a different layer in a multi-layered sampling scheme as shown in Figure 1. By combining all layer values in a full factorial scheme, all design options are subjected to the same uncertainties and a direct comparison for several future scenarios is enabled.

This multi-layered Monte Carlo scheme concept is the basis of the global probabilistic design method presented in Van Gelder et al. (2014), as shown in Figure 2. It is combined with sensitivity analysis, meta-modelling, sampling efficiency and convergence control (Janssen, 2013). This method contains four steps: preprocessing, preliminary screening, updating and the actual probabilistic design. Note that Figure 2 illustrates the use of only one scenario layer, but adding more scenario layers, as is done in this paper case study, can be done analogously.

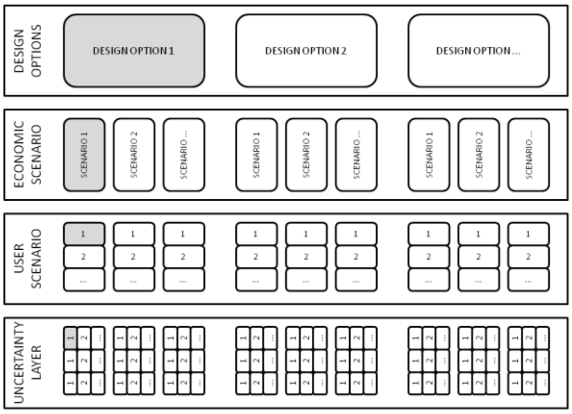


Figure 1 Multi-layered sampling scheme. First Monte Carlo run is indicated in grey.

Methodology

The design problem is first preprocessed to select the needed output parameters and a suitable simulation model. Input parameters are determined and fixed values or input distributions are ascribed for respectively deterministic and stochastic parameters.

In the preliminary screening, meta-models are constructed and validated, as presented in Figure 2 and also explained in Van Gelder et al. (2013b). Meta-models mimic the original, potentially time-intensive model with a simpler and faster surrogate model. As the proposed multi-layered sampling scheme requires execution of numerous Monte Carlo simulations, which may easily become computationally (too) expensive, the use of these meta-models might be very interesting. The training and validation sets therefore needed, are also used to calculate sensitivity indices to rank the input parameters from most to least influencing the output distributions.

Based on this sensitivity ranking, the distributions of most influencing parameters can be updated, while the less influencing parameters can be omitted. Limiting the number of parameters eases collecting the required input distributions, as this might be time-consuming. Moreover, this improves sampling efficiency and limits the number of considered design options in the multi-layered scheme.

In the probabilistic design step (see Figure 2), first all potential design options are chosen. Then both uncertainty and scenario parameters are independently sampled to create an initial multi-layered scheme. To start the Monte Carlo loop, the first design option and first scenario are selected. The initial uncertainty sample is run in the simulation model and is enlarged until the desired outputs are converged. These outputs are sufficiently converged when adding samples does not change their values more than a user defined percentage. After that, the next scenario values are analogously run and more values can be added until convergence of the design option or until all potential scenario values are calculated. Then, one can continue with the next

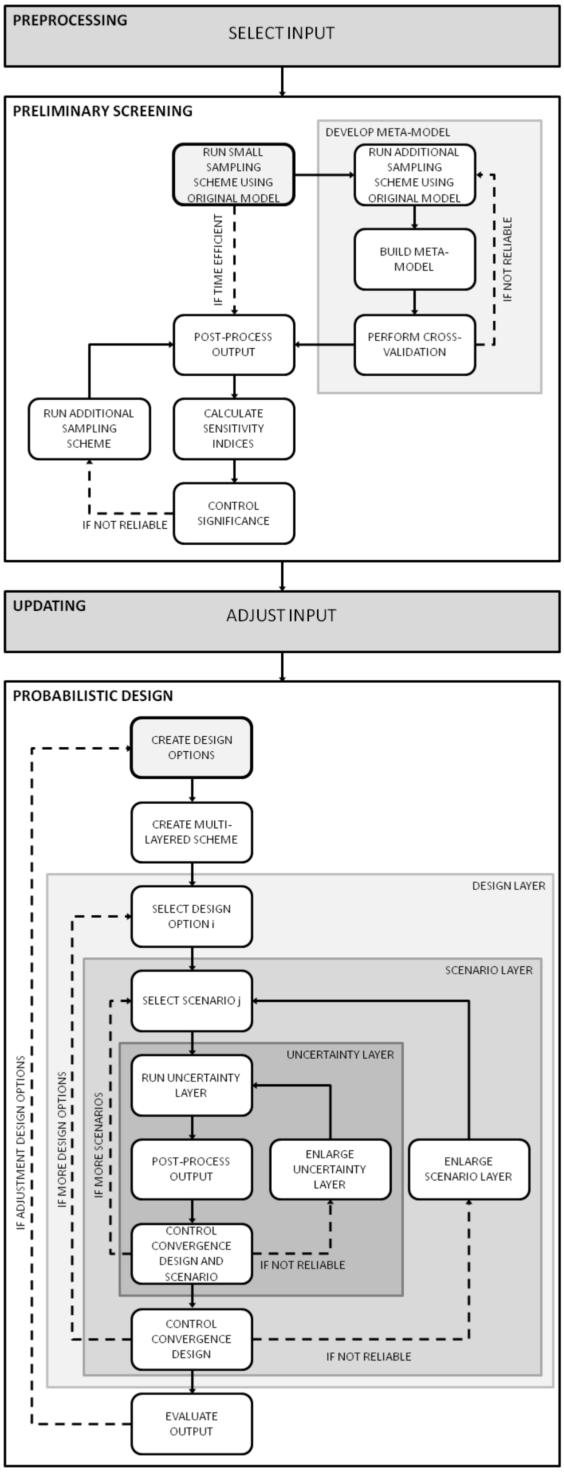


Figure 2 Flowchart probabilistic design

design option. If all design options are converged, the outputs can be evaluated.

Robust design output evaluation

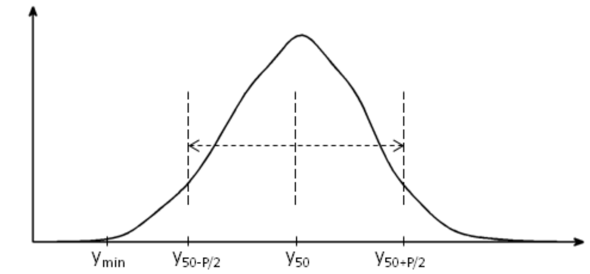
As mentioned in the introduction, dwelling owners need confidence in the net present costs of their investments in energy efficiency. Ideas from robust design are therefore incorporated by optimising mean performance and minimising spread (Zang et al., 2005). That way, designs that best resist the uncertain parameters can be selected. For that purpose, effectiveness ε and robustness R_p indicators were defined and illustrated in previous research (Van Gelder et al., 2013a). For a positive output parameter y to be minimised, the indicators for a specific future scenario are (see Figure 3):

$$\varepsilon(x_i, s) = 1 - \frac{y_{50}(x_i,s) - y_{\min}}{y_{50} - y_{\min}} \qquad (1)$$

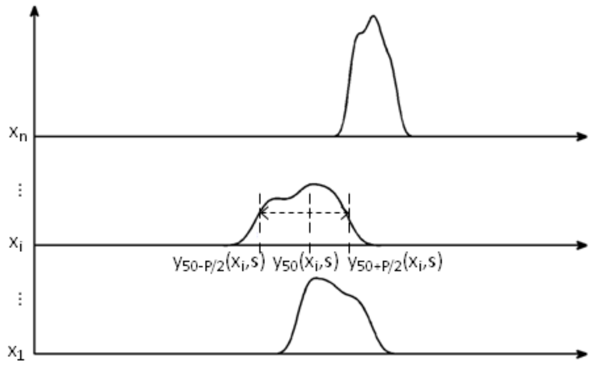
$$R_P(x_i, s) = 1 - \frac{y_{50+P/2}(x_i,s) - y_{50-P/2}(x_i,s)}{y_{50+P/2} - y_{50-P/2}} \qquad (2)$$

with y_q the qth percentile of y under full uncertainty of all parameter categories, $y_q(x_i,s)$ the qth percentile after selecting a future scenario s and design option x_i and P the user specified percentage of included sample points. y_{min} corresponds to the minimal calculated y value which is not an outlier, whereby an outlier is defined as a sample point smaller than y_{25}-$1.5(y_{75}$-$y_{25})$.

Effectiveness ε thus describes how the deviation between median performance and optimal performance (y_{min}) for a certain design and scenario improves compared to the design under full uncertainty. Robustness R_p is analogously determined as the improvement the performance spread of a design option makes in proportion to the spread under full uncertainty. According to these definitions, a solution with an effectiveness and robustness of one is the best possible, while negative values are to be avoided.



(a) *PDF of output parameter value y under full uncertainty*



(b) PDF of output parameter value y for design options x_i and considered scenario s

Figure 3 Probability density functions (PDF) of output parameter value y under full uncertainty and after selection of design option x_i and scenario s

SIMULATION

A cost optimisation of a typical Flemish dwelling geometry is performed to illustrate how the proposed probabilistic design method can be used in decision-making. Building envelope characteristics, ventilation technologies and sun shading are combined, and of these combinations, the economically most effective and robust low-energy dwelling designs will be selected based on Pareto-optimality. To overcome overheating problems in these dwellings, those designs with an overheating risk will be excluded. As different user types and energy price evolutions are considered in this study, the aim is to obtain overall optimal and thus robust solutions.

Case study

To illustrate robust dwelling design, an average Flemish dwelling is chosen as case study. The semi-detached dwelling, as shown in Figure 4, might be a typical dwelling in a social housing neighbourhood with a mix of future inhabitants. It has a floor area of 140 m², an uninsulated basement, and overhangs for sun shading. Several low-energy design options are compared to help the housing company in selecting the most cost-effective and cost-robust options, with a comfortable indoor climate as additional constraint. Therefore, both energy demand and maximal temperature are simulated with a dynamic BES model, replaced by a meta-model to reduce calculation time, and net present costs are calculated afterwards.

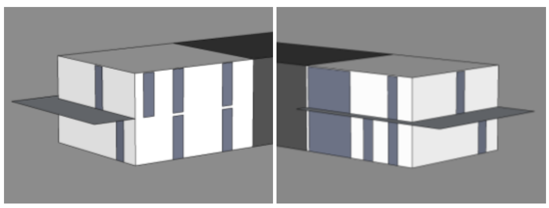


Figure 4 Dwelling model

Dynamic BES model

The dwelling is modelled with two thermal zones and simulated in a transient BES tool developed in Modelica (Baetens et al., 2012) for the reference climate year of Uccle, Belgium (Van Gelder et al., 2013a). The adjacent dwelling is considered at a constant temperature of 19 °C. To simulate the heat demand, an ideal heating system is assumed, which is controlled using simplified occupancy and temperature profiles. A ventilation system is incorporated in the model with or without heat recovery. In summer, the heating system and heat recovery are switched off. To optimise the summer comfort, if the day zone temperature exceeds the user dependent comfort temperature, the air change rate is doubled for the next six hours or until the occupants leave the dwelling. This algorithm simulates the user behaviour to help achieving a comfortable indoor climate.

Cost calculation

After the heat demands are computed, the corresponding net present costs over 30 years are calculated with a cost calculation tool developed in research project IWT TETRA BEP2020 (Verbeeck et al., 2013), following the European standard EN ISO 15459. This means that the energy costs for heating and ventilation and investment and maintenance costs of only energy-related dwelling components are taken into account. Energy-related costs that are the same for all considered design options, such as the heating system and household electricity, are disregarded in this study as well.

Note that for the total net present cost, also the disregarded investment costs, such as heating system, masonry and foundation, corresponding maintenance costs and energy costs for electricity and domestic hot water would have to be taken into account.

Input parameters BES model

The probabilistic parameters taken into account in the BES model are listed in Table 1 according to their parameter category. For each parameter, several low-energy design values are taken into account.

Six types of ventilation systems are implemented: A, A+, C, C+, D and D+. The labelling corresponds to Belgian standard NBN D 50-001 (1991), where natural ventilation is indicated with A, mechanical exhaust ventilation with C and mechanical balanced ventilation with D. The '+' indicates the presence of occupant detection. The air change rate is then lowered when occupants are absent. Type D and D+ are equipped with heat recovery.

Based on commercially available glazing types, five types of windows are considered with different U- and g-values to vary heat losses and solar gains through the windows.

Sunscreens are implemented as well. There are five possibilities: no sunscreens, sunscreens on the south facade with a transmission of 10 % or 30 % or sunscreens on all facades with a transmission of 10 % or 30 %. The sunscreens are controlled manually or automatically.

User behaviour is known to be very influential for the maximal temperature and energy demand. Therefore, several user parameters are considered based on a measurement campaign in 70 new dwellings in Flanders (Belgium) (Staepels et al., 2013). These parameters are attributed to the user scenario.

The measurement campaign (Staepels et al., 2013) indicated that indoor air quality is very variable, supposing that in only a part of the dwellings the nominal ventilation rate is reached. Based on these findings, Weibull distributions are proposed.

Note that for clarity, in this case study, many other parameters are considered deterministic, such as

Table 1 Probabilistic parameters BES model

	PARAMETER	DISTRIBUTION*
DESIGN	Infiltration rate n_{50}	Uni(0.44, 12.3) /h
	Ventilation system	Dis(A, A+, C, C+, D, D+)
	Heat recovery	Uni(0.7, 0.95)
	U-value wall	Uni(0.1, 0.3) W/m²K
	U-value roof	Uni(0.1, 0.3) W/m²K
	U-value floor	Uni(0.1, 0.3) W/m²K
	Construction type	Dis(massive, timberframe)
	Window type	Dis(2.07 W/m²K & g=0.613, 2.07 W/m²K & g = 0.512, 1.29 W/m²K & g = 0.631, 1.31 W/m²K & g = 0.551, 0.7 W/m²K & g = 0.407)
	Sunscreen type	Dis(none, 10 %, 10 % south, 30 %, 30 % south)
	Sunscreen control	Dis(manual, automatic 1, automatic 2, automatic 3)
USER SCENARIO	Occupancy day zone	Dis(1, 2, 3, 4)
	Occupancy night zone	Dis(1, 2, 3)
	Set temperature occupancy day zone	Nor(21, 1.35) °C
	Set temperature absence day zone	Dis(15, no reduction) °C
	Set temperature occupancy night zone	Nor(19, 2) °C
	Internal heat gains persons	Uni(35, 175) W
	Internal heat gains basic	Uni(20, 180) W
	Internal heat gains appliances summer	Uni(130, 1000) W
	winter	Uni(180, 1300) W
	autumn/spring	Uni(140, 1150) W
UNC.	Air change rate day zone	Wei(0.6576, 4.67) /h
	night zone	Wei(1.7847, 4.67) /h

* Explanation of the symbols used:

Dis(a,b,c): discrete distribution with equal probability for a, b and c

Nor(μ,σ): normal distribution wit mean value μ and standard deviation σ

Uni(a,b): uniform distribution with equal probability between a and b

Wei(λ,k): Weibull distribution with scale factor λ and shape factor k climate and delivery efficiency of the heating system. Because the focus lies on the probabilistic approach, they are not described in this paper.

Input parameters cost calculation

As explained earlier, the energy demand obtained from the dynamic BES-model is used as input in the calculation tool. Furthermore, as we are interested in the net present costs of energy measures, the nominal energy price evolution is of major interest. Three discrete values are considered, inspired by previous price evolutions: -1.5 %, 2.3 % and 10 %.

Analogously to the BES model inputs, other parameters are considered deterministic, such as investment and maintenance costs.

DISCUSSION AND RESULT ANALYSIS

As described earlier in this paper, an optimisation is performed of the net present cost effectiveness and robustness of a typical dwelling in for example a social housing neighbourhood. Potential future scenarios are taken into account to obtain overall optimal designs.

Preprocessing

Input and output parameters and a simulation model were selected as described in the previous section.

Preliminary screening

In this paper, the dynamic BES model is replaced by a meta-model to calculate the heat demand and maximal temperature of the design options. Training and validation sets of the parameters in Table 1 are therefore run in the BES model. These sets are then mimicked with cubic multivariate adaptive regression splines (MARS) (Friedman, 1991, Jin et al., 2001, Jekabsons 2011) because of their good approximation ability and their fast calculation (Van Gelder et al., 2013b). Due to the use of hinge functions, model complexities can easily be taken into account. MARS models are usually of the form

$$\underline{y} = \sum_{i=1}^{m} c_i B_i(x) \tag{3}$$

with $\underline{y}$ the estimated output parameter, x the input parameter vector, m the amount of basis functions B_i, which can be a constant, a hinge function or a product of two hinge functions and c_i the weight factors.

With these sampling sets, sensitivity indices were calculated with Spearman's rank correlation as explained in Van Gelder et al. (2013a). The ventilation system, set temperatures, infiltration rate, window type, internal heat gains, ventilation rate and U-value of the walls are most influencing the net energy demand. The construction type, sunscreen type, window type and internal heat gains have the most impact on the maximal temperature. Furthermore, the construction type, sunscreen type, set temperatures, ventilation type, infiltration rate and window type are most influencing the net present cost. The nominal energy price evolution has of course a high influence on this net present cost as well. U-values of roofs and floors are thus least influencing considered output parameters.

Updating

To reduce calculation time, U-values of floors and roofs are omitted in the probabilistic design as they appeared to be least dominant. Average values are thus considered in the further research. One can consider to omit the U-value of the walls and the ventilation rate as well as they are only significantly influencing the heat demand, which is not considered as criteria in the design problem.

Because the infiltration rate and heat recovery seemed to be very important for the net present cost, workmanship errors on these design parameters are added.

Probabilistic design

First, the design options are selected based on the sensitivity analysis results and the multi-layered scheme is created as shown in Table 2. For each *design parameter*, several low-energy design values are taken into account. All meaningful combinations of these design values result in 10.800 design options. The nominal energy price evolution and user type are taken into account as *scenario parameters*. The users are quantified based on different set temperatures, occupancy profiles and internal heat gains. By considering the *scenario parameters*, we are able to study the optimal results for each of the nine potential evolution and user type combinations. In the multi-layered scheme of Figure 1 and Table 2, 100 uncertainty layer values are sampled in sets of 20 with a *maximin* Latin Hypercube scheme (Husslage et al., 2008). This number is sufficient for convergence of this case study. For simplicity in this paper, every design option and scenario combination is subjected to the same 100 samples, resulting in 9.720.000 calculation combinations.

After the multi-layered scheme is created, it can be calculated with the constructed meta-models and cost calculation tool. With the described meta-model, these calculations can be done in a few hours. For each of the nine scenario combinations, this results in a cumulative distribution function (CDF) for the net present cost and the maximal temperature for each design option as shown in Figure 5 and Figure 6 for one of the scenarios. To avoid overheating risks, those design options where the indoor temperature may rise above 28 °C are penalised in each scenario. These design options are indicated in grey in Figure 5 and Figure 6.

Finally, the outputs can be evaluated to select the best performing design options. For each scenario combination, effectiveness ε and robustness R_{95} are calculated according to Eq. (1) and (2), as plotted in Figure 7. Of all these design options, the individual

Table 2 Multi-layered scheme

	PARAMETER	DISTRIBUTION*
DESIGN	Infiltration rate n_{50}	Dis(0.6, 1, 3) /h
	Ventilation system (and heat recovery)	Dis(A, A+, C, C+, D 70 %, D 80 %, D 90 %, D+ 70 %, D+ 80 %, D+ 90 %)
	U-value wall	Dis(0.1, 0.15, 0.18, 0.24) W/m²K
	U-value roof	0.2 W/m²K
	U-value floor	0.2 W/m²K
	Construction type	Dis(massive, timberframe)
	Window type	Dis(2.07 W/m²K & g=0.613, 2.07 W/m²K & g = 0.512, 1.29 W/m²K & g = 0.631, 1.31 W/m²K & g = 0.551, 0.7 W/m²K & g = 0.407)
	Sunscreen type	Dis(none, 10 %, 10 % south, 30 %, 30 % south)
	Sunscreen control	Dis(manual, automatic)
SCENARIO	Nominal energy price evolution	Dis(-1.5 %, 2.3 %, 10 %)
	User type	Dis(saving, average, wasting)
UNCERTAIN	Air change rate	
	day zone	Wei(0.6576, 4.67) /h
	night zone	Wei(1.7847, 4.67) /h
	Workmanship error infiltration rate	Nor(1, 0.1)
	Workmanship error heat recovery	Nor(1, 0.1)

* Explanation of the symbols used:

Dis(a,b,c): discrete distribution with equal probability for a, b and c

Nor(μ,σ): normal distribution wit mean value μ and standard deviation σ

Wei(λ,k): Weibull distribution with scale factor λ and shape factor k

Pareto front is then calculated and indicated in blue on Figure 7. Depending on the user type and economic scenario, other Pareto optimal solutions are found.

The energy price evolution is highly influencing the net present cost, as the scenario value has an high impact on the robustness and effectiveness. The user type seems to most influence the overheating, because of the internal heat gains. Therefore, more energy saving measures, such as a low infiltration rate and lower U-values are more optimal for high cost increases and more expensive sunscreens are

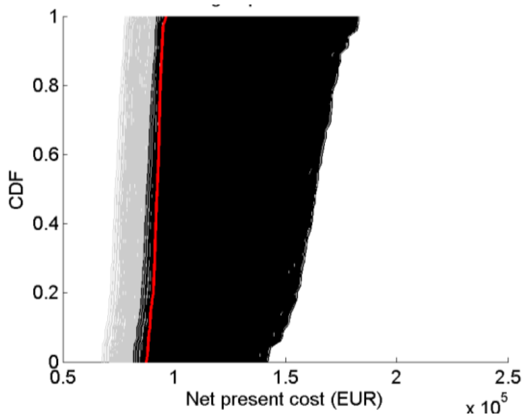


Figure 5 Cumulative distribution functions of net present cost for all design options for an average user and an energy price evolution of 10 %. Options with an overheating potential are indicated in grey.

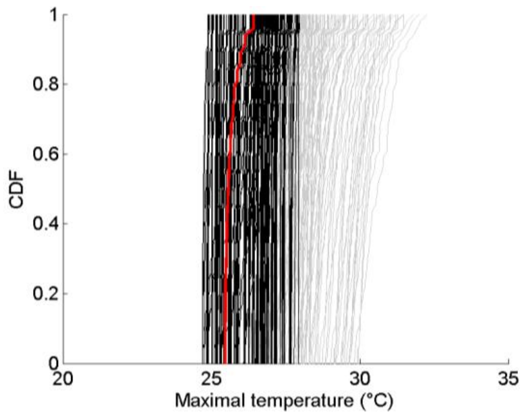


Figure 6 Cumulative distribution functions of maximal temperature for all design options for an average user and an energy price evolution of 10 %. Options with an overheating potential are indicated in grey.

more optimal for wasting users. A massive construction and balanced ventilation systems are common in all Pareto fronts. Only for users of the type 'saving' or 'average' and an energy cost reduction, a natural ventilation system can be optimal. However, these options are less robust. Some timber-framed solutions appear in the Pareto fronts as well, but they are less effective because the investment costs are higher and more or better sunscreens are needed.

Since the dwelling owners cannot impose user behaviour and economic evolutions, it is better to apply scenario-independent measures. Therefore, the overall Pareto optimal solutions are calculated. All design options with an overheating risk in one of the scenarios are penalised. As we are interested in the common design options in the upper right corners of Figure 7, all design options with an effectiveness ε lower than -0.2 and robustness R_{95} lower than 0.7 in at least one of the scenario combinations are penalised as well. Of the remaining design options,

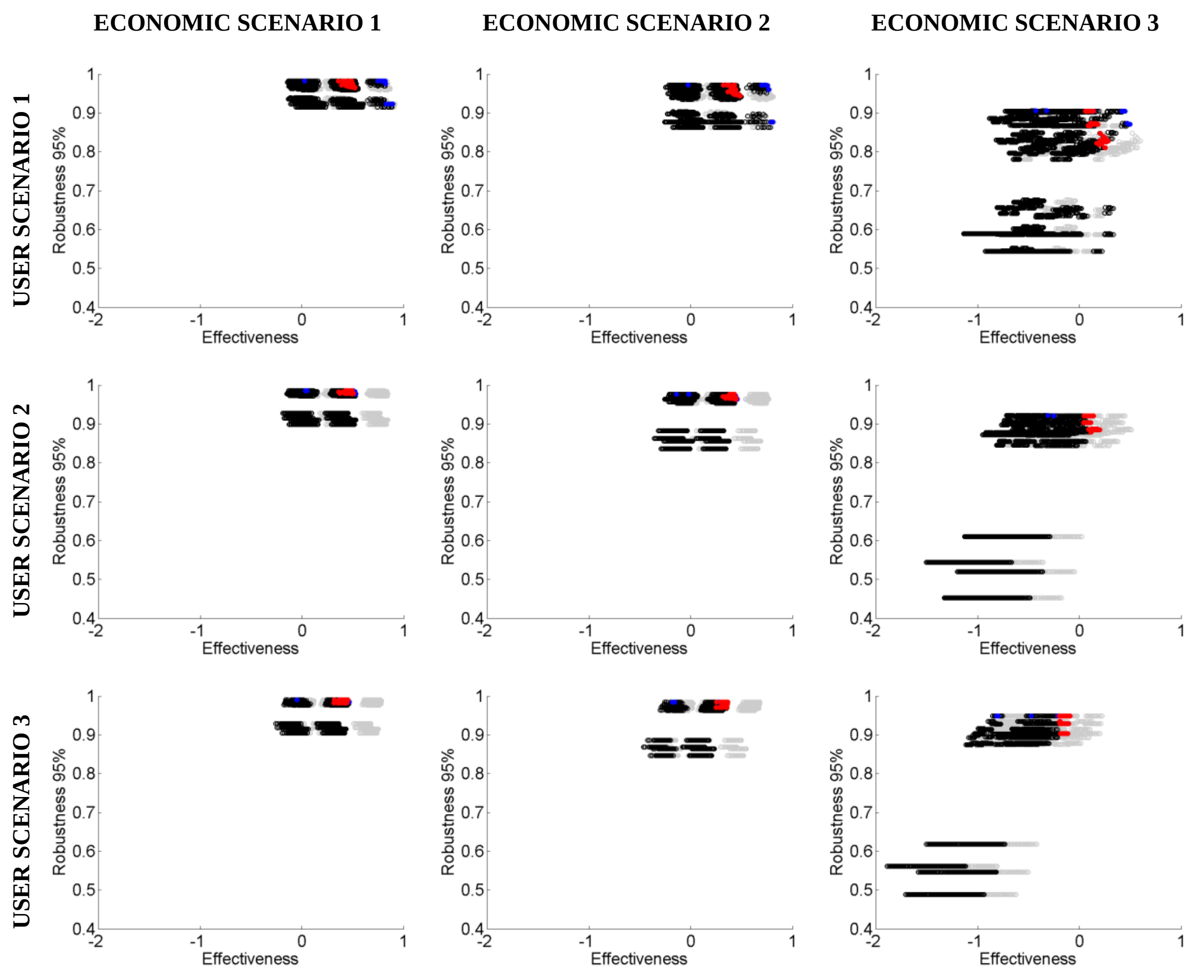


Figure 7 Robustness in function of effectiveness of net present cost for all design options for the nine considered user type and price evolution scenarios. Individual Pareto optimal solutions are indicated in blue, while the overall Pareto optimal solutions are indicated in red.

an overall Pareto front of effective and robust design options is calculated. The optimisation criteria therefore used are all effectivenesses ε and robustnesses R_{95} of the nine considered scenario combinations. The obtained overall 18-dimensional Pareto front is indicated in red in Figure 7. To reduce the net present cost and its spread - regardless of what happens in the future - most important design values are a balanced ventilation system with occupancy detection, a massive construction, high-performing windows and basic sunscreens. One of the overall Pareto front solutions is indicated in red in Figure 5 and Figure 6 to illustrate the optimal design options. This design option is indeed effective, as it has a low net present cost. It is also robust because of its relative low spread and the maximal temperature is lower than 28 °C. Because the overall Pareto front is performing very well in all potential scenarios and is quite independent of the inherently uncertain parameters, the net present cost of these design options can be reliably predicted with a small range. These Pareto options are thus most interesting for the dwelling owner.

In this paper, only one performance, i.e. the net present cost, is optimised and the maximal temperature is used as penalising output. Of course, this can be easily expanded to more performance criteria, such as investment costs, energy use or CO_2 emission, in a multi-objective optimization. Instead of using Pareto-optimality, the weighted sum method can be used as well to optimise multiple effectiveness and robustness indicators. This allows attaching more importance to some performance parameters or to only effectiveness or robustness. Weight factors can also be used in considering the probability of occurrence of the scenario combinations. The proposed methodology is thus very effective and flexible.

CONCLUSIONS

This paper presented a probabilistic design method based on multi-layered Monte Carlo schemes, meta-modelling and sensitivity analysis, as introduced in Van Gelder et al. (2014). The method was used to design a cost-effective and cost-optimal low-energy dwelling, while overheating was avoided. Comparing 10.800 design options for three economic and three user scenarios needed 9.720.000 calculations, which could be done in a few hours because of a time-efficient meta-model. This emphasizes the efficacy of the proposed design method in comparing both effectiveness and robustness of multiple performance criteria for numerous design options.

In order to create effective and robust designs for the net present cost, a massive construction with sunscreens, windows with low U-values and balanced ventilation with occupancy detection and heat recovery is preferred. In each of the potential future economic and user scenarios, these design options perform well, while other design options might result in undesired performances. This stresses the importance of such a probabilistic design in order to promote robust building solutions.

ACKNOWLEDGEMENT

The authors are very thankful for the funding of the Flemish government and companies for the IWT TETRA BEP2020 project (Verbeeck et al., 2013) and participating inhabitants for the related measurement campaign. Many thanks to Liesbeth Staepels to develop the cost calculation tool according to the European standard EN ISO 15429. They would like to thank Bart Husslage and Gijs Rennen from the Tilburg University as well for sharing their MATLAB code for calculation of maximin sampling schemes (Husslage et al., 2008).

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